

Reconciling the LEP and SLAC measurements of $\sin^2 \theta_w$

Francesco Caravaglios^{a*},
Graham G. Ross^{b†},

^aDepartment of Physics, Theoretical Physics,
University of Oxford, 1 Keble Road, Oxford OX1 3NP

^b Theory Division, CERN, CH-1211, Geneva, Switzerland

Abstract

We consider whether a discrepancy between the SLAC and LEP measurements of $\sin^2 \theta_w$ can be explained by new physics. We find that only the contribution of a new neutral gauge boson, Z' , nearly degenerate with the Z can affect the SLAC measurement while leaving the LEP observables almost unaffected. We briefly discuss possible signals for this new gauge boson, including changes in the Z lineshape when measured with polarised electrons, small changes in R_b , A_{FB}^e , and larger changes in two jet and $t\bar{t}$ production at hadron colliders.

Introduction

In the context of the Standard Model, the value of $\sin^2 \theta_w$ determined by SLAC [1] from the measurement of the A_{LR} asymmetry currently disagrees at the 2.5 standard deviation level with the value obtained from a variety of precision measurements performed at the LEP collider. All channels at the LEP experiments[2] give a value of the Weinberg angle ($\sin^2 \theta_w = 0.2321 \pm 0.0004$) which is consistent with the Standard Model prediction [3][4][5] ($\sin^2 \theta_w = 0.2320$) for a top mass of about 174 GeV [6]. On the other hand the A_{LR} asymmetry measured at SLAC gives $\sin^2 \theta_w = 0.2292 \pm 0.001$ requiring a much heavier top quark for consistency with the Standard Model.

The immediate question raised by this discrepancy is whether it signals new physics beyond the Standard Model. In this letter we will discuss the nature of the new physics

*INFN Fellow

†SERC Senior Fellow, on leave from ^a

that can allow the two measurements to be consistent. We will show that the only possibility is the existence of a new neutral gauge boson, Z' , whose mass and coupling to the fermions is strongly constrained. Moreover this new gauge boson may be responsible for the small excess in the LEP measurement of $R_b = \Gamma_b/\Gamma_h = 0.2192 \pm 0.0018$ [2] compared to the Standard Model Prediction [7][3] $R_b = \Gamma_b/\Gamma_h = 0.2157 \pm 0.0005$ ($M_{top} = 175 \pm 15$ GeV, $M_H = 300$ GeV). Finally we consider possible tests for such a new gauge boson.

Constraints on the new physics

The possibility of explaining the difference between the LEP and SLAC measurements through new physics arises because they refer to different observables:

- SLAC uses polarized initial beams to measure the LR asymmetry

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \quad (1)$$

where σ_L and σ_R are respectively the cross sections (at the Z_0 peak) of

$$\begin{aligned} e_L + \bar{e}_L &\longrightarrow X \\ e_R + \bar{e}_R &\longrightarrow X \end{aligned} \quad (2)$$

where $e_{L(R)}$ is a lefthanded (righthanded) electron and X is an hadronic or a $\tau^+\tau^-$ final state. The value measured at SLAC is

$$A_{LR}^{SLAC} = 0.163 \pm 0.0079 \quad (3)$$

- LEP uses unpolarized initial beams to study the asymmetries of a fermion-antifermion pair in the final state. Within the Standard Model we may use the value of $\sin^2 \theta_w$ obtained at LEP (averaged over all channels) to predict the LR asymmetry (henceforth we will call A_{LR}^{LEP} the *prediction* of LEP measurements for the SLAC asymmetry)

$$A_{LR}^{LEP} = 0.142 \pm 0.0032 \quad (4)$$

In the Standard Model A_{LR}^{SLAC} and A_{LR}^{LEP} should be equal but new physics can change the expectation for one or both. However (with the possible exception of R_b which is discussed below), the consistency of all LEP precision measurements with the Standard Model for a top mass of about 174 GeV [6] means that changing A_{LR}^{LEP} significantly (via a change of $\sin^2 \theta_w$) is unacceptable¹[8].

Thus we must look for new physics that changes A_{LR}^{SLAC} while leaving A_{LR}^{LEP} LEP measurements essentially unchanged. In particular the accuracy of the LEP measurement for the total unpolarised hadronic cross section (plus $\tau^+ \tau^-$ events)

$$\frac{\sigma_L + \sigma_R}{2} = (43.49 \pm 0.12) \text{ nb} \quad (LEP) \quad (5)$$

¹A particularly clear case is the LEP measurement of A_{FB}^e : as will be obvious from eq(10) with $q=e$, it is not possible to *decrease* A_{FB}^e to the value 0.0156 [2] obtained if one uses the SLAC result $g_V^e/g_A^e = 0.082$.

is so precise (and in agreement with the Standard Model prediction) that we must require the changes $\delta\sigma_{R,L}$ in $\sigma_{L,R}$ to satisfy

$$\delta\sigma_R + \delta\sigma_L \simeq 0 \quad (6)$$

to the accuracy of eq(5). This accuracy is much better than the discrepancy between eqs.(3) and (4). Hence the requirement that the theoretical prediction for A_{LR}^{SLAC} be increased means that the new physics must give $\delta\sigma_R \approx -\delta\sigma_L$ and $\delta\sigma_R/\sigma_R \simeq -(1 \div 2)\%$.

The new contribution needed to generate $\delta\sigma_R < 0$ must come from an interference between the Standard Model amplitude and the amplitude coming from the new physics because a non-interfering term would necessarily give $\delta\sigma_R > 0$. The Standard Model amplitude for the process of eq(2) has the form

$$M_0^{R,L} = a\bar{u}(p_q)\gamma^\mu(g_V^q + g_A^q\gamma_5)v(q_q) \times \bar{v}(p_e)\gamma_\mu(g_V^e + g_A^e\gamma_5) \left(\frac{1 \pm \gamma_5}{2}\right) u(q_e) \quad (7)$$

corresponding to the processes (2) with $q^2 = M_Z^2$. The quantity a is determined by the Z propagator and on resonance is purely imaginary. In order to interfere with this amplitude the new physics must generate an amplitude, $\delta M^{L,R}$, of the form

$$\delta M^{L,R} = \delta a\bar{u}(p_q)\gamma^\mu(g_V^{q'} + g_A^{q'}\gamma_5)v(q_q) \times \bar{v}(p_e)\gamma_\mu(g_V^{e'} + g_A^{e'}\gamma_5) \left(\frac{1 \pm \gamma_5}{2}\right) u(q_e) \quad (8)$$

with δa imaginary². The squared matrix element is

$$|M_0^{L,R} + \delta M^{R,L}|^2 = |M_0^{R,L}|^2 + 2Re(M_0^{L,R}\delta M^{L,R*}) + |\delta M^{L,R}|^2. \quad (9)$$

The first term is the Standard Model contribution, and we assume for the moment the third one to be small compared with the others. The second term is the interference between the Z_0 contribution and the new physics contribution. In the processes measured at LEP we must take the average over the initial polarizations giving (ignoring fermion masses)

$$\begin{aligned} \sum_{L,R} Re(M_0^{L,R}\delta M^{L,R*}) &= Re(a\delta a^* Tr(\gamma^\mu \not{p}_e \gamma^\nu \not{q}_e ((g_V^e g_V^{e'} + g_A^e g_A^{e'}) + (g_V^e g_A^{e'} + g_V^{e'} g_A^e)\gamma_5))) \\ &\quad \times Tr(\gamma_\mu \not{p}_q \gamma_\nu \not{q}_q ((g_V^q g_V^{q'} + g_A^q g_A^{q'}) + (g_V^q g_A^{q'} + g_V^{q'} g_A^q)\gamma_5)) \end{aligned} \quad (10)$$

If the predictions for the LEP measurements in this channel labelled by q are to remain essentially those of the Standard Model we must require

$$(g_V^e g_V^{e'} + g_A^e g_A^{e'}) \simeq 0 \quad and \quad (g_V^q g_A^{q'} + g_A^q g_V^{q'}) \simeq 0 \quad (11)$$

or

$$(g_V^e g_A^{e'} + g_A^e g_V^{e'}) \simeq 0 \quad and \quad (g_V^q g_V^{q'} + g_A^q g_A^{q'}) \simeq 0. \quad (12)$$

If either of these conditions holds one of the traces in (10) has the term proportional to γ_5 vanishing, while the other has only the term proportional to γ_5 non-vanishing. Thus the

²Neglecting real terms which do not interfere is a simplification which does not alter the conclusions of the following discussion because they are strongly constrained by the requirement that LEP measurements be unaffected to the accuracy of eq(5).

vanishing of the interference (10) may be immediately seen because one term is symmetric in (μ, ν) while the other is antisymmetric. Further by definition either of the relations in eqs(11) or eq(12) imply $\delta\sigma_L(\theta) \simeq -\delta\sigma_R(\theta)$ where θ is the centre of mass scattering angle. Finally we note that the relation (11) implies $\delta\sigma_R(\theta)$ is even in $\cos(\theta)$ while the relation (12) implies that it is odd so that only the former allows for a non- zero amplitude integrated over θ . Putting all this together we conclude that the new physics must generate an amplitude of the form of eq(8) with a imaginary and further constrained by eq(11). Only in this case can one change the prediction for the SLAC asymmetry measurement while leaving the predictions for the LEP measurements unchanged.

However this is possible only for a restricted class of final states X . In the case $X = e^+e^-$ it is impossible to satisfy eq(11) and moreover the additional non-Z contribution is positive. Thus to preserve the agreement of the LEP measurements with the Standard Model we must keep the couplings of the Z' to the electron small. In the case $X = \tau^+\tau^-$ the consistency of the LEP τ polarisation measurements together with the unpolarised τ measurements require eq(11) be satisfied for $g_V^{q,q'} = \pm g_A^{q,q'}$, $q = \tau$ (i.e. purely left-handed and right-handed final states). These can only be satisfied if the coupling of the new physics to the τ is small. We note that the τ constraints are satisfied if we assume universal lepton couplings and if the electron constraints discussed above are satisfied.

Nature of the new physics

We turn now to a discussion of the nature of the physics beyond the Standard Model capable of generating such a matrix element. The discussion above implies that the new physics is predominantly coupled to the quark sector. We will consider the following three possibilities which arise at tree or one loop level:

- i). The new physics generates a correction to the Z coupling as in Fig 1a.
- ii). The new physics generates a new box contribution as in Fig 1b.
- iii). The new physics generates a new contribution at tree level via a Born graph.

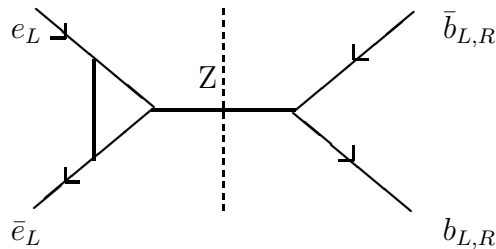


Fig 1a

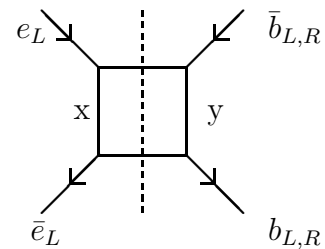


Fig 1b

Let us consider the first case. It should be noticed that the matrix elements $\delta\bar{M}_{L,R}$ (we call $\delta\bar{M}_{L,R}$ the matrix element $\delta M_{L,R}$ after imposing the condition eq(11)) cannot be generated by new physics contributing a Feynman diagram with the Z coupled to fermions through a vertex loop diagram (Fig 1a). The measurement of the polarisation asymmetry

at SLAC is sensitive only to modifications of the electron vertex as in Fig. 1a. As discussed above such a non-standard Z_0 -electron effective vertex coupling introduced to explain the A_{LR}^{SLAC} would necessarily increase the A_{FB}^e measured at LEP. (This conclusion applies even if one adds a box diagram contribution as well as its correction is also always positive, as it is evident from eq(10) (with the index $q \rightarrow e$.) Thus we conclude mechanism i). cannot reconcile LEP and SLAC.

The second possibility is that the interference is due to the imaginary part of a box diagram involving states beyond the Standard Model as in Fig 1b.

However a more detailed analysis of the conditions (11) shows that such a term does *not* introduce a matrix element of the type $\delta\bar{M}_{L,R}$. We know the value $g_V^e/g_A^e = 1 - 4\sin^2\theta_w$ and $g_V^b/g_A^b = 1 - 4/3\sin^2\theta_w$ within the Standard Model (for simplicity we have taken $q = \text{bottom}$; the conclusions are the same for the other quarks). Using this in eq(11) implies $|g_A^b| > |g_V^b|$. This means that the imaginary part of the box diagrams $\delta\bar{M}_{L,R}$ of the two processes

$$\begin{aligned} e_L + \bar{e}_L &\longrightarrow b_L + \bar{b}_L \\ e_L + \bar{e}_L &\longrightarrow b_R + \bar{b}_R \end{aligned} \tag{13}$$

must have opposite signs. To see that this is not possible note that the only terms that can change sign between the two processes come from the propagators X and Y (the remainder of the graph comes in two complex conjugate parts with a definite sign). However these propagators are both spacelike and hence the sign is independent of the identity of the states X and Y which may change for the two processes of eq(13). Thus we see the sign of the box diagrams of the two processes (13) must be the same and they can never give the correct $|g_A^b| > |g_V^b|$. (The argument may be generalised to more complicated higher loop graphs.)

Thus we are left with option iii). as the only possible source of a matrix element of the type δM_1 is the exchange of a new gauge boson, Z' . Provided it is produced nearly on resonance its amplitude will be largely imaginary as desired. Unlike the box graphs in this case can one have opposite signs for $\delta\sigma_L$ and $\delta\sigma_R$ simply through the choice of the Z' couplings. In the next section we consider in detail whether such a new contribution can indeed explain the discrepancy between LEP and SLAC.

Numerical analysis

Here we consider the couplings of the Z' needed to change the peak observables. As stressed above its coupling to the electron must be small compared to the Z in the channel $e^+ + e^- \rightarrow e^+ + e^-$. On the other hand we need a measureable contribution in the channel $e^+ + e^- \rightarrow q + \bar{q}$, so we need a sizeable coupling to a quark. We start by assuming the new Z' couples only to the b (and t quarks). Including (small) non-interference effects there are three experimental measurements sensitive to such a new Z' contribution namely $A_{LR}^{SLAC} = 0.163 \pm 0.0079$, $R_b = 0.2192 \pm 0.0018$ and $A_{FB}^b = 0.0967 \pm 0.0038$ [2][4]. These

are determined by the three independent parameters³ $g_A^{e'}/g_V^{e'}, g_V^{b'}/g_A^{b'}$ and δa ⁴. A fit (with $M_{top} = 175 \text{ GeV}$ and $M_{Higgs} = 300 \text{ GeV}$) gives⁵

$$\begin{aligned}\frac{g_A^{e'}}{g_V^{e'}} &= -0.046 \pm 0.18 \\ \frac{g_V^{b'}}{g_A^{b'}} &= -0.716 \pm 0.03 \\ \frac{\delta a}{a} &= -0.13 \pm 0.05.\end{aligned}\tag{14}$$

Here we have arranged that the new neutral Z' simultaneously explains the discrepancy between LEP and SLAC and the small excess over the Standard Model prediction in R_b . Since the values of eq(14) nearly satisfy eq(11) the contribution of the interference term in eq(9) is comparable to the non-interference term and the discrepancy between the predicted and measured values of R_b is of the minimum order to be expected from the need to explain the discrepancy between the SLAC and CERN results. However this expectation is not absolute as it is possible to find a fit consistent with the Standard Model result for R_b by fixing $g_A^{e'}/g_V^{e'}$ and $g_V^{b'}/g_A^{b'}$ so the discrepancy is not a firm prediction of the new neutral current.

Tests of a new neutral gauge boson

We have shown that the SLAC and LEP results may only be reconciled through a new Z' gauge boson nearly degenerate with the Z . In practice this means that the Z' should lie within $\Gamma_Z + \Gamma_{Z'}$ of the Z mass. The most obvious test of this possibility will be forthcoming when SLAC measure the asymmetry off the Z peak for only in the case of exact degeneracy of the Z and Z' will the line shape remain unchanged. What about further tests? By construction the most significant effects have been put in the b quark sector and we have seen that this can lead to observable deviations from the Standard Model. However this depends on the precise choice of $g_A^{e'}/g_V^{e'}$ and $g_V^{b'}/g_A^{b'}$ and does not provide a definitive test.

However Eq(14) strongly constrains the relative Z' couplings to the electron channel. The matrix elements $|M_0|^2$ and $|\delta M_1|^2$ are given by

$$|M_0|^2 \sim \frac{\Gamma_e \Gamma_b}{\Gamma_Z^2} \quad ; \quad |\delta M_1|^2 \sim \frac{\Gamma_e' \Gamma_b'}{\Gamma_{Z'}^2}\tag{15}$$

which allow us to estimate⁶

$$\frac{\Gamma_e'}{\Gamma_b'} \simeq 0.8 \times 10^{-4}.\tag{16}$$

³The other parameters can always be reabsorbed into a redefinition of this three ones.

⁴ δa and a are defined setting $g_V^{e'} = 1/2$ and $g_A^{b'} = 1/2$, and $g_A^e = g_A^b = 1/2$

⁵Here we assume that all the R_b data come only from Z_0 -peak. Including off peak data of R_b needs the knowledge of both the Z_0 and Z' lineshapes.

⁶Here we assume the dominance of the b -channel $\Gamma_b'/\Gamma_{Z'} \simeq 1$.

As we have already noted even if the coupling of the Z' to the electron is very small, because the interference between the matrix elements of Z and Z' does not vanish when both final and initial states are electrons, there may be significant effects in this channel. We can use the fit (14) and the ratio (16) to predict the effects to the $e^+ + e^- \rightarrow e^+ + e^-$ channel. While the interference does not affect the total cross section (this may be seen to follow from the predominantly vector-like nature of the new current to the electron) the effect to the forward-backward asymmetry is

$$\delta A_{FB}^e = 0.004^{+0.0039}_{-0.0026}. \quad (17)$$

Thus a Z' coupled principally to the b quark can be detected from a precision measurement of the forward-backward asymmetry of the electron (in the case of lepton universality also the forward-backward asymmetries of the τ and the μ are similarly affected⁷, while there are negligible effects in the τ -polarization measurements).

Of course the fit of eq(14) depends on the assumption that only the b, \bar{b} final states are affected by this Z' . If we assume that more quarks are (equally) coupled to this new gauge boson the electron coupling will be reduced by the number of quark couplings assumed⁸. In this case the effects on R_b , A_{FB}^b and A_{FB}^e will similarly be reduced. It will also lead to an excess of two jet production near the Z_0 mass in the $p\text{-}\bar{p}$ colliders (UA2 gives $\sigma = 9.6 \pm 2.3(stat) \pm 1.1(syst) \text{ nb}$ which is only slightly above the Standard model prediction (5.8 nb) [9] - this constrains the Z' couplings to light quarks in this case to be comparable to the Z couplings.). Similarly a Z' coupling to top quarks will also enhance $t\bar{t}$ production, again going in the direction favoured by current experimental measurements.

We conclude that the left-right asymmetry measurement of SLAC is compatible with all LEP measurements *only* if we assume the existence of a Z' with resonant contribution which overlaps the Z_0 lineshape⁹. Signals of such a gauge boson could come from small deviations from the Standard Model predictions for R_b , A_{FB}^e or from larger deviations in hadron colliders giving enhanced two jet production (if the Z' is coupled to light quarks) or enhanced $t\bar{t}$ production (if the Z' is coupled to top quarks). Further tests are available at SLAC for the prediction is that R_b should change (if the Z' is coupled to b quarks) for different initial electron polarization and, more definitively, the polarised line shape should vary due to the different interference pattern expected off resonance.

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⁷We note that the current experimental value (with $M_{top} = 174 \text{ GeV}$ and $M_{Higgs} = 300$) is $\delta A_{FB}^e = 0.0 \pm 0.0034$ and $\delta A_{FB}^l = 0.002 \pm 0.0016$ (with lepton universality).

⁸Note that the absolute magnitude of the Z' coupling is not determined as we have assumed it to be produced on resonance.

⁹We have deliberately avoided adding any theoretical prejudice on the possible nature of the new physics but it must be admitted that it is difficult to provide a convincing theoretical argument leading to such a degeneracy in Z, Z' masses.

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